Recall basic Cons. of Energy...

$$\rho c\left(\frac{\partial T}{\partial t} + \underline{v}\circ\nabla T\right) = \underbrace{k\nabla^2 T}_{\text{or } \nabla\circ(k\nabla T)} + U'''_{gen}$$

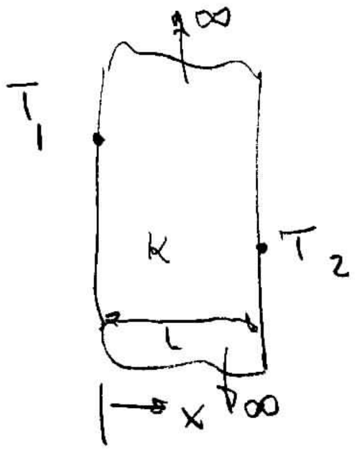
Assumes
$\rho, c, k = \text{¢}$

For S.S., $\underline{V} = \underline{0}$, $U'''_{gen} = 0$ ... $k\nabla^2 T = 0$

Now pick your coordinate system.

Cartesian $\quad \nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$

- For a 1-D wall of thicknes L



What is $T(x)$?

Note: T is assumed to only vary in x-direction

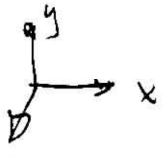
$$\frac{d^2 T}{dx^2} = 0$$

$\int$ once...

$$\frac{dT}{dx} = \text{¢}_1$$

$\int$ once

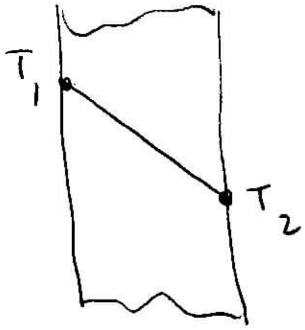
$$T = \text{¢}_1 x + \text{¢}_2$$



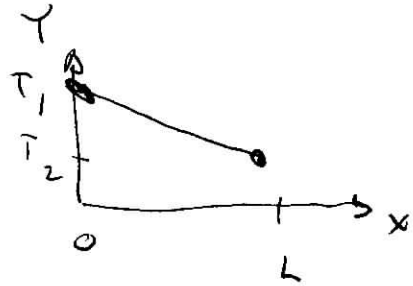
Use B.C.

$$T\big|_{x=0} = \text{¢}_2 = T_1$$

$$T\big|_{x=L} = \text{¢}_1 L + T_1 = T_2 \Rightarrow \text{¢}_1 = (T_2 - T_1)/L$$

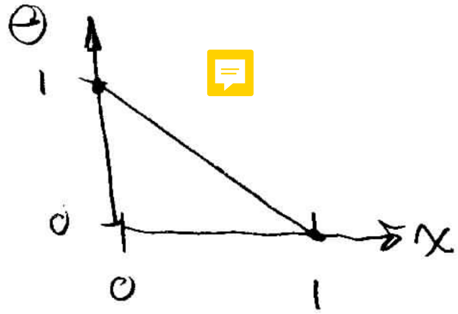


$$T = \frac{(T_2 - T_1)}{L}\, x + T_1$$



or

$$\underbrace{\frac{T - T_1}{T_2 - T_1}}_{\equiv \Theta} = \underbrace{\frac{x}{L}}_{\equiv \chi}$$



$$\boxed{\Theta = \chi}$$

But recall

$$q = -kA\frac{\partial T}{\partial n} = -kA\frac{dT}{dx}$$

$$= -kA\frac{(T_2 - T_1)}{L}$$

$$= \left(\frac{kA}{L}\right)(T_2 - T_1)$$

Compare

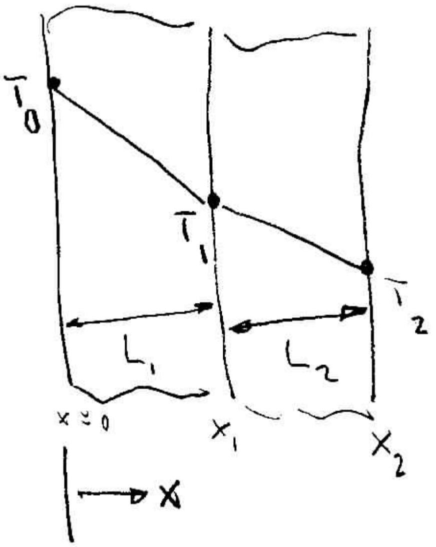
$V = IR$

$\Delta T = q R_{therm}$

or $$q = \frac{(T_2 - T_1)}{R_{therm}}$$

so $\boxed{R = L/kA}$ for wall

- How about 2 walls touching...



$$q = \frac{T_1 - T_0}{R_1} \qquad q = \frac{T_2 - T_1}{R_2}$$

$$T_1 - T_0 = q R_1 \qquad T_2 - T_1 = q R_2$$

Add these equations

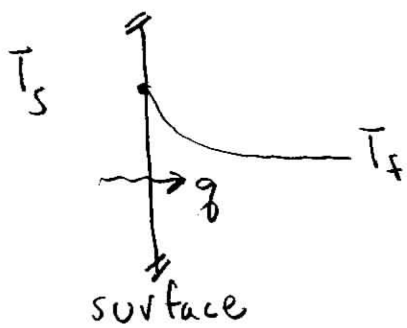
$$(T_1 - T_0) + (T_2 - T_1) = q R_1 + q R_2$$

$$T_2 - T_0 = q (R_1 + R_2)$$

$$\boxed{\Delta T_{overall} = q \sum R_i}$$

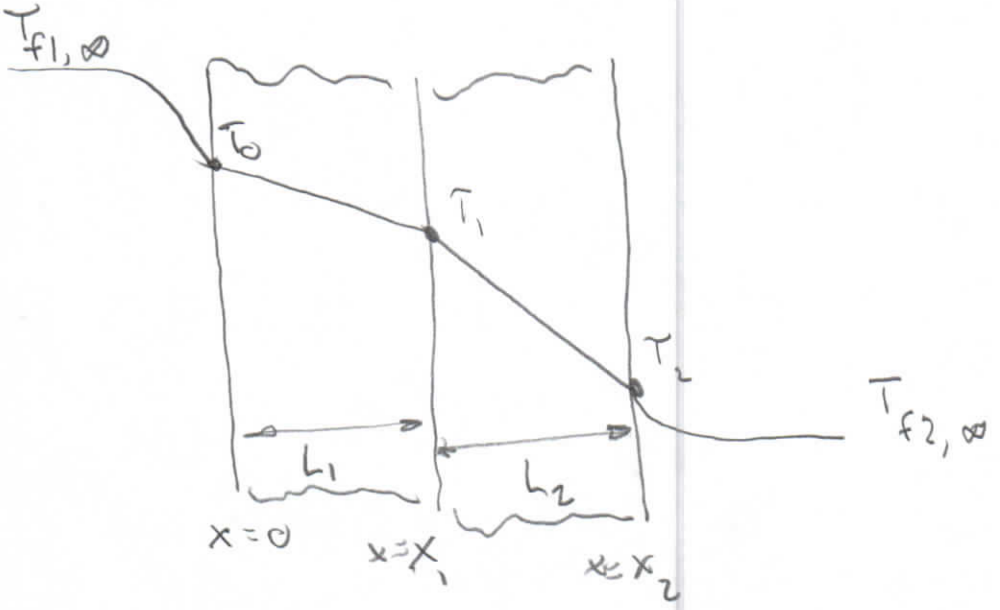
- How about 2 touching wall, each of which is in contact with a moving fluid...

First recall



$$q = \underbrace{h}_{\text{fluid conv. heat transfer coef.}} A (T_s - T_f) = \frac{T_s - T_f}{R_{conv}}$$

$$\Rightarrow R_{conv} = \frac{1}{hA}$$



$$q = \frac{T_0 - T_{1,\infty}}{R_{conv,1}},\quad \frac{T_1 - T_0}{R_1},\quad \frac{T_2 - T_1}{R_2},\quad \frac{T_{2,\infty} - T_2}{R_{conv,2}}$$

so

$$T_0 - T_{1,\infty} = R_{conv,1}\, q \qquad T_1 - T_0 = q R_1 \qquad T_2 - T_1 = q R_2 \qquad T_{2,\infty} - T_2 = q R_{conv,2}$$

Add these

$$(T_0 - T_{1,\infty}) + (T_1 - T_0) + (T_2 - T_1) + (T_{2,\infty} - T_2) = q\left(R_{conv,1} + R_1 + R_2 + R_{conv,2}\right)$$

$$\underbrace{T_{2,\infty} - T_{1,\infty}}_{\Delta T_{overall}} = q \sum R_i$$

where $R_{total} = \dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A} + \dfrac{L_2}{k_2 A} + \dfrac{1}{h_2 A}$

$$= \frac{1}{A}\underbrace{\left(\frac{1}{h_1} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_2}\right)}_{\text{call this } 1/U} \equiv \frac{1}{AU}$$